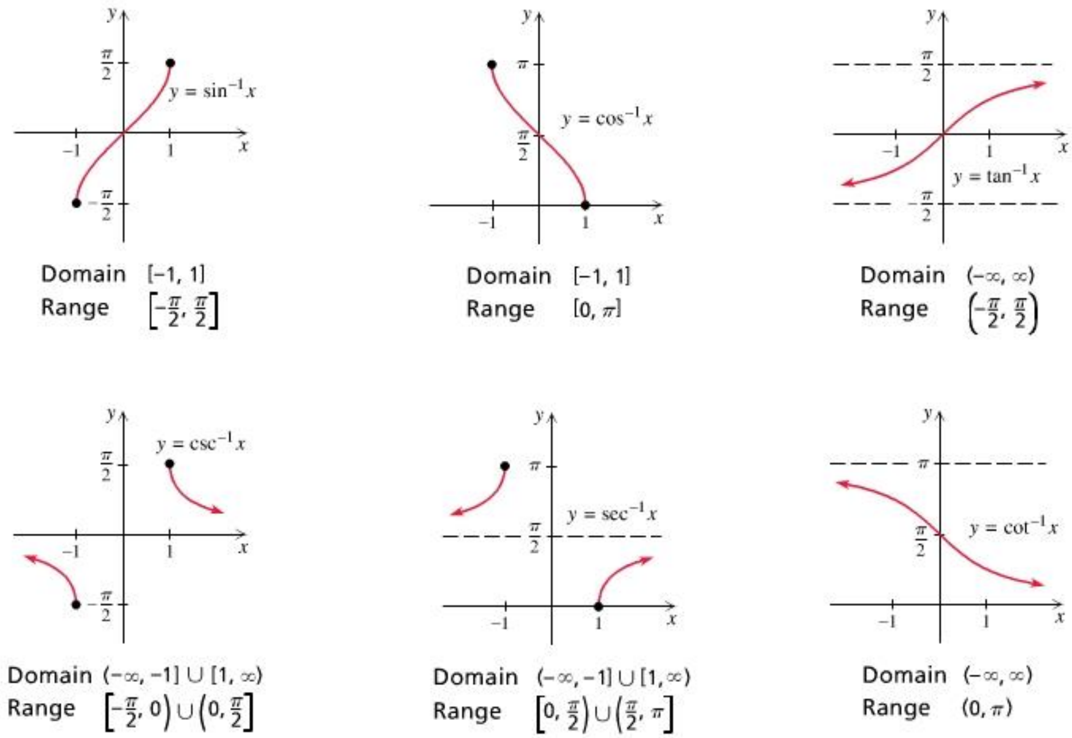
The Inverse Trigonometric Functions

In order for a function to have an inverse, the function must be one-to-one. In other words, it must pass the horizontal line test. Since the trigonometric functions are periodic, we must restrict the domain so that they will pass the horizontal line test. Therefore, the inverse functions will have a restricted range.



The **inverse sine function** is sometimes called the **arc sine**, and is abbreviated $\arcsin(x)$ or $\sin^{-1}(x)$. Similarly, the other inverse functions are often called the **arc cosine** and **arc tangent**, abbreviated $\arccos(x)$ or $\cos^{-1}(x)$ and $\arctan(x)$ or $\tan^{-1} x$.

$\sin^{-1}(x)$ is the angle in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ whose sine is *x*.

$\cos^{-1}(x)$ is the angle in $[0, \pi]$ whose cosine is *x*.

$\tan^{-1}(x)$ is the angle in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is *x*.

Example: Find the exact value of each expression without using a table or calculator.

a) $\sin^{-1}\left(\frac{1}{2}\right)$

b) $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$

c) $\tan^{-1}(1)$

d) $\arccos\left(-\frac{1}{2}\right)$

Example: Find the angle α.

a) $\sin\alpha = 0.56,\ -90° \le \alpha \le 90°$

b) $\tan\alpha = -3,\ -\pi/2 < \alpha < \pi/2$

c) $\cos\alpha = 0.23,\ 0° \le \alpha \le 180°$

d) $\cos\alpha = -0.82,\ 0 \le \alpha \le \pi$

Inverses of the Reciprocal Trigonometric Functions

$\csc^{-1}(x)$ is the angle in $\left[-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right]$ whose cosecant is *x*.

$\sec^{-1}(x)$ is the angle in $\left[0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right]$ whose secant is *x*.

$\cot^{-1}(x)$ is the angle in $(0,\pi)$ whose cotangent is *x*.

Identities

$$\csc^{-1}(x) = \sin^{-1}\left(\frac{1}{x}\right)$$

$$\sec^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right)$$

$$\cot^{-1}(x) = \begin{cases} \tan^{-1}(1/x) & \text{for } x > 0 \\ \tan^{-1}(1/x) + \pi & \text{for } x < 0 \\ \pi/2 & \text{for } x = 0 \end{cases}$$

$$\cot^{-1}(x) = \frac{\pi}{2} - \tan^{-1}(x)$$

Example: Find the exact value of each expression without using a table or calculator.

a) $\operatorname{arcsec}(-2)$

b) $\csc^{-1}(2)$

c) $\operatorname{arccot}\left(\frac{1}{\sqrt{3}}\right)$

Example: Find the approximate value of each expression rounded to 4 decimal places.

a) $\operatorname{arccsc}(-1.4713)$

b) $\cot^{-1}(-2.5)$

c) $\sec^{-1}(4.328)$

Example: Find the exact value of each composition.

a) $\sin\left(\cot^{-1}(-1)\right)$

b) $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$

c) $\arcsin\left(\cos\left(\frac{\pi}{6}\right)\right)$

Examples: Find the exact value of each composition.

a) $\sin\left(\cos^{-1}\left(\frac{8}{17}\right)\right)$

b) $\cot\left(\arccos\left(\frac{5}{13}\right)\right)$

Example: Find an equivalent algebraic expression for $\sin(\arctan(x))$

Example: Find an equivalent algebraic expression for $\sin(\operatorname{arccot}(x))$